# 6.7 Graph Linear Inequalities in Two Variables

**F.a.1** Describe, recognize, interpret and translate graphical representations of mathematical and real-world phenomena on coordinate grids . . .

**Before** You graphed linear equations in two variables.

**Now** You will graph linear inequalities in two variables.

**Why?** So you can analyze a music competition, as in Ex. 56.

**Key Vocabulary**
- **linear inequality in two variables**
- **graph of an inequality in two variables**

A **linear inequality in two variables**, such as $x - 3y < 6$, is the result of replacing the = sign in a linear equation with $<$, $\leq$, $>$, or $\geq$. A **solution of an inequality in two variables** $x$ and $y$ is an ordered pair $(x, y)$ that produces a true statement when the values of $x$ and $y$ are substituted into the inequality.

## EXAMPLE 1 Standardized Test Practice

**Which ordered pair is *not* a solution of $x - 3y \leq 6$?**

Ⓐ $(0, 0)$ Ⓑ $(6, -1)$ Ⓒ $(10, 3)$ Ⓓ $(-1, 2)$

**Solution**

Check whether each ordered pair is a solution of the inequality.

**Test (0, 0):**

$x - 3y \leq 6$ Write inequality.

$0 - 3(0) \leq 6$ Substitute 0 for $x$ and 0 for $y$.

$0 \leq 6$ ✓ Simplify.

**Test (6, −1):**

$x - 3y \leq 6$ Write inequality.

$6 - 3(-1) \leq 6$ Substitute 6 for $x$ and $-1$ for $y$.

$9 \leq 6$ ✗ Simplify.

So, $(0, 0)$ is a solution of $x - 3y \leq 6$ but $(6, -1)$ is *not* a solution.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

## GUIDED PRACTICE for Example 1

**Tell whether the ordered pair is a solution of $-x + 2y < 8$.**

**1.** $(0, 0)$ **2.** $(0, 4)$ **3.** $(3, 5)$

**GRAPH OF AN INEQUALITY** In a coordinate plane, the **graph of an inequality in two variables** is the set of points that represent all solutions of the inequality. The *boundary line* of a linear inequality divides the coordinate plane into two **half-planes**. Only one half-plane contains the points that represent the solutions of the inequality.

## KEY CONCEPT *For Your Notebook*

### Graphing a Linear Inequality in Two Variables

**STEP 1** **Graph** the boundary line. Use a *dashed line* for $<$ or $>$, and use a *solid line* for $\leq$ or $\geq$.

**STEP 2** **Test** a point not on the boundary line by checking whether the ordered pair is a solution of the inequality.

**STEP 3** **Shade** the half-plane containing the point if the ordered pair is a solution of the inequality. Shade the other half-plane if the ordered pair is *not* a solution.

## EXAMPLE 2 Graph a linear inequality in two variables

**Graph the inequality $y > 4x - 3$.**

**Solution**

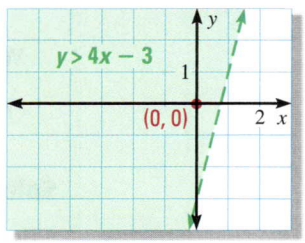
**STEP 1** **Graph** the equation $y = 4x - 3$. The inequality is $>$, so use a dashed line.

**STEP 2** **Test** $(0, 0)$ in $y > 4x - 3$.

$$0 \stackrel{?}{>} 4(0) - 3$$

$$0 > -3 \checkmark$$

**STEP 3** **Shade** the half-plane that contains $(0, 0)$, because $(0, 0)$ is a solution of the inequality.




## EXAMPLE 3 Graph a linear inequality in two variables

**Graph the inequality $x + 2y \leq 0$.**

**Solution**

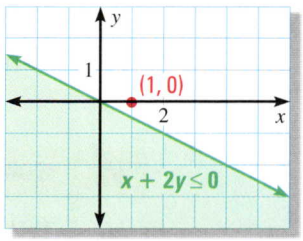
**STEP 1** **Graph** the equation $x + 2y = 0$. The inequality is $\leq$, so use a solid line.

**STEP 2** **Test** $(1, 0)$ in $x + 2y \leq 0$.

$$1 + 2(0) \stackrel{?}{\leq} 0$$

$$1 \leq 0 \times$$

**STEP 3** **Shade** the half-plane that does not contain $(1, 0)$, because $(1, 0)$ is *not* a solution of the inequality.




**AVOID ERRORS**

Be sure to test a point that is not on the boundary line. In Example 3, you can't test $(0, 0)$ because it lies on the boundary line $x + 2y = 0$.

## GUIDED PRACTICE for Examples 2 and 3

**4.** Graph the inequality $x + 3y \geq -1$.

**LINEAR INEQUALITIES IN ONE VARIABLE** The steps for graphing a linear inequality in two variables can be used to graph a linear inequality in one variable in a coordinate plane.

The boundary line for an inequality in one variable is either vertical or horizontal. When testing a point to determine which half-plane to shade, do the following:

- If an inequality has only the variable $x$, substitute the $x$-coordinate of the test point into the inequality.
- If an inequality has only the variable $y$, substitute the $y$-coordinate of the test point into the inequality.

## EXAMPLE 4 Graph a linear inequality in one variable

**Graph the inequality $y \geq -3$.**

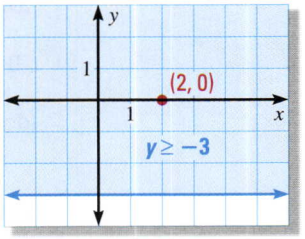
**Solution**

***STEP 1*** **Graph** the equation $y = -3$.
The inequality is $\geq$, so use a solid line.

***STEP 2*** **Test** (2, 0) in $y \geq -3$. You substitute only the $y$-coordinate, because the inequality does not have the variable $x$.

$$0 \geq -3 \checkmark$$

***STEP 3*** **Shade** the half-plane that contains (2, 0), because (2, 0) is a solution of the inequality.




## EXAMPLE 5 Graph a linear inequality in one variable

**Graph the inequality $x < -1$.**

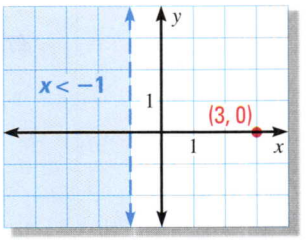
**Solution**

***STEP 1*** **Graph** the equation $x = -1$.
The inequality is $<$, so use a dashed line.

***STEP 2*** **Test** (3, 0) in $x < -1$. You substitute only the $x$-coordinate, because the inequality does not have the variable $y$.

$$3 < -1 \times$$

***STEP 3*** **Shade** the half-plane that does *not* contain (3, 0), because (3, 0) is not a solution of the inequality.




Animated Algebra at classzone.com

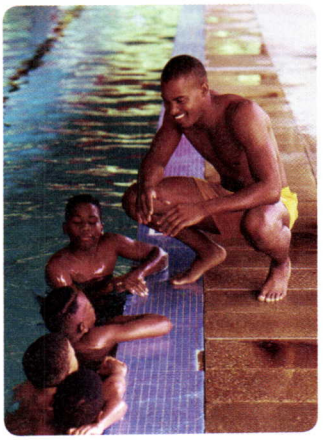
## GUIDED PRACTICE for Examples 4 and 5

**Graph the inequality.**

**5.** $y > 1$ **6.** $y \leq 3$ **7.** $x < -2$

## EXAMPLE 6 Solve a multi-step problem

**JOB EARNINGS** You have two summer jobs at a youth center. You earn \$8 per hour teaching basketball and \$10 per hour teaching swimming. Let $x$ represent the amount of time (in hours) you teach basketball each week, and let $y$ represent the amount of time (in hours) you teach swimming each week. Your goal is to earn at least \$200 per week.



- Write an inequality that describes your goal in terms of $x$ and $y$.
- Graph the inequality.
- Give three possible combinations of hours that will allow you to meet your goal.

### Solution

**STEP 1** **Write** a verbal model. Then write an inequality.

| Basketball pay rate (dollars/hour) | · | Basketball time (hours) | + | Swimming pay rate (dollars/hour) | · | Swimming time (hours) | ≥ | Total earnings (dollars) |
|---|---|---|---|---|---|---|---|---|
| ↓ | | ↓ | | ↓ | | ↓ | | ↓ |
| 8 | · | $x$ | + | 10 | · | $y$ | ≥ | 200 |

**AVOID ERRORS**
The variables can't represent negative numbers. So, the graph of the inequality does not include points in Quadrants II, III, or IV.

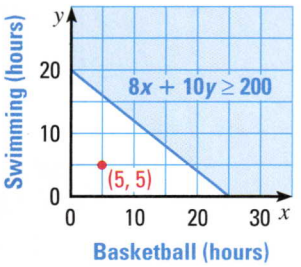
**STEP 2** **Graph** the inequality $8x + 10y \geq 200$.

First, graph the equation $8x + 10y = 200$ in Quadrant I. The inequality is ≥, so use a solid line.

Next, test (5, 5) in $8x + 10y \geq 200$:

$$8(5) + 10(5) \geq 200$$

$$90 \geq 200 \text{ ✗}$$




Finally, shade the part of Quadrant I that does not contain (5, 5), because (5, 5) is not a solution of the inequality.

**STEP 3** **Choose** three points on the graph, such as (13, 12), (14, 10), and (16, 9). The table shows the total earnings for each combination of hours.

| Basketball time (hours) | 13 | 14 | 16 |
|---|---|---|---|
| Swimming time (hours) | 12 | 10 | 9 |
| Total earnings (dollars) | 224 | 212 | 218 |

## GUIDED PRACTICE for Example 6

8. **WHAT IF?** In Example 6, suppose that next summer you earn \$9 per hour teaching basketball and \$12.50 per hour teaching swimming. Write and graph an inequality that describes your goal. Then give three possible combinations of hours that will help you meet your goal.

# 6.7 EXERCISES

**HOMEWORK KEY**

○ = **WORKED-OUT SOLUTIONS** on p. WS15 for Exs. 5, 19, and 57

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 39, 56, 59, and 60

◆ = **MULTIPLE REPRESENTATIONS** Ex. 55

## SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The ordered pair $(2, -4)$ is a(n) ___?___ of $3x - y > 7$.

2. ★ **WRITING** *Describe* the difference between graphing a linear inequality in two variables and graphing a linear equation in two variables.

**EXAMPLE 1** on p. 405 for Exs. 3–15

**CHECKING SOLUTIONS** **Tell whether the ordered pair is a solution of the inequality.**

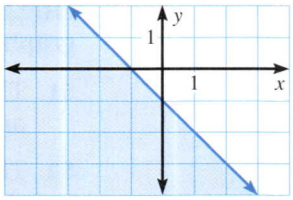
3. $x + y < -4$; $(0, 0)$
4. $x - y \leq 5$; $(8, 3)$
5. $y - x > -2$; $(-1, -4)$
6. $2x + 3y \geq 14$; $(5, 2)$
7. $4x - 7y > 28$; $(-2, 4)$
8. $-3y - 2x < 12$; $(5, -6)$
9. $2.8x + 4.1y \leq 1$; $(0, 0)$
10. $0.5y - 0.5x > 3.5$; $(6, 2)$
11. $x \geq -3$; $(-4, 0)$
12. $y \leq 8$; $(-9, -7)$
13. $\frac{3}{4}x - \frac{1}{3}y < 6$; $(-8, 12)$
14. $\frac{2}{5}x + y \geq 2$; $(1, 2)$

15. ★ **MULTIPLE CHOICE** Which ordered pair is *not* a solution of $x + 5y < 15$?

Ⓐ $(-1, -3)$ Ⓑ $(-1, 3)$ Ⓒ $(1, 3)$ Ⓓ $(3, 2)$

**EXAMPLES 2, 3, 4, and 5** on pp. 406–407 for Exs. 16–38

16. ★ **MULTIPLE CHOICE** The graph of which inequality is shown?




Ⓐ $x + y \leq -1$ Ⓑ $x + y \geq -1$

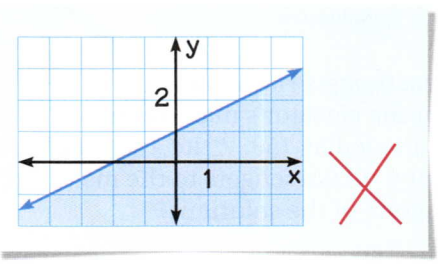
Ⓒ $x - y \leq -1$ Ⓓ $x - y \geq -1$

**GRAPHING INEQUALITIES** **Graph the inequality.**

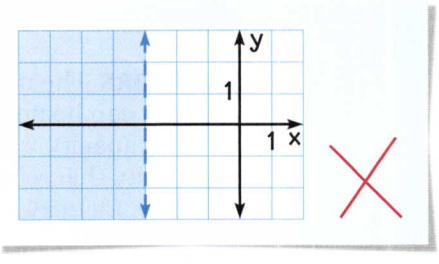
17. $y > x + 3$
18. $y \leq x - 2$
19. $y < 3x + 5$
20. $y \geq -2x + 8$
21. $x + y < -8$
22. $x - y \leq -11$
23. $x + 8y > 16$
24. $5x - y \geq 1$
25. $2(x + 2) > 7y$
26. $y - 4 < x - 6$
27. $-4y \leq 16x$
28. $6(2x) \geq -24y$
29. $y < -3$
30. $x \geq 5$
31. $x > -2$
32. $y \leq 4$
33. $3(x - 2) > y + 8$
34. $x - 4 \leq -2(y + 6)$
35. $\frac{1}{2}(x + 2) + 3y < 8$
36. $2(x + 1) \geq \frac{1}{4}y - 1$

**ERROR ANALYSIS** ***Describe*** **and correct the error in graphing the inequality.**

37. $2y - x \geq 2$




38. $x \leq -3$



**39.** ★ **WRITING** Can you use (0, 0) as a test point when graphing $2x > -5y$? *Explain* your reasoning.

**TRANSLATING SENTENCES** **Write the verbal sentence as an inequality. Then graph the inequality.**

**40.** Four less than $x$ is greater than or equal to $y$.

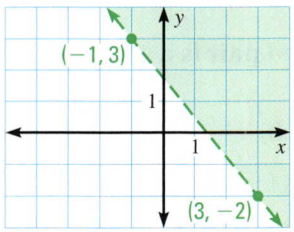
**41.** The product of $-2$ and $y$ is less than or equal to the sum of $x$ and 6.

**42.** The quotient of $y$ and 2 is greater than the difference of 7 and $x$.

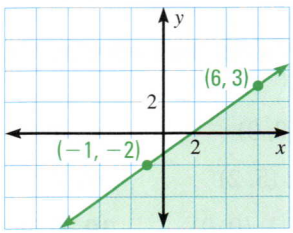
**43.** The sum of $x$ and the product of 4 and $y$ is less than $-3$.

**USING A GRAPH** **Write an inequality of the graph shown.**

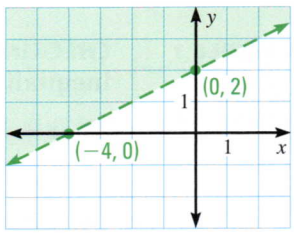
**44.**




**45.**




**46.**




**WRITING INEQUALITIES** **Write an inequality whose graph contains only the points in the given quadrants.**

**47.** Quadrants I and II

**48.** Quadrants II and III

**49.** Quadrants III and IV

**50.** Quadrants I and IV

**CHALLENGE** **In Exercises 51 and 52, write and graph an inequality whose graph is described by the given information.**

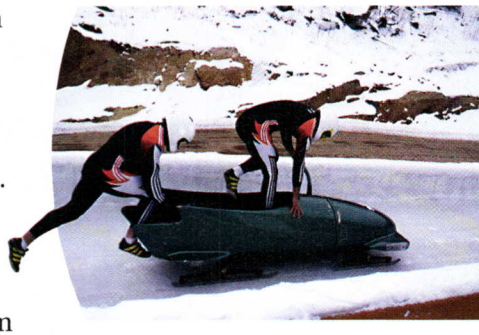
**51.** The points (2, 5) and $(-3, -5)$ lie on the boundary line. The points (6, 5) and $(-2, -3)$ are solutions of the inequality.

**52.** The points $(-7, -16)$ and (1, 8) lie on the boundary line. The points $(-7, 0)$ and (3, 14) are *not* solutions of the inequality.

## PROBLEM SOLVING

**EXAMPLE 6** on p. 408 for Exs. 53–57

**53.** **BOBSLEDS** In a two-man bobsled competition, the sum of the weight $x$ (in pounds) of the bobsled and the combined weight $y$ (in pounds) of the athletes must not exceed 860 pounds. Write and graph an inequality that describes the possible weights of the bobsled and the athletes. Identify and interpret one of the solutions.



@HomeTutor for problem solving help at classzone.com

**54.** **ELEVATORS** The number $y$ of passengers riding an elevator can be no greater than the elevator's maximum weight capacity $x$ (in pounds) divided by 150. Write and graph an inequality that relates the number of passengers to the maximum weight capacity. Identify and interpret one of the solutions.

@HomeTutor for problem solving help at classzone.com



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

**55.** **MULTIPLE REPRESENTATIONS** You tutor Spanish for $15 per hour and French for $10 per hour. You want to earn at least $100 per week.

**a. Writing an Inequality** Write an inequality that describes your goal in terms of hours spent tutoring Spanish and hours spent tutoring French.

**b. Drawing a Graph** Graph the inequality. Then give three possible combinations of hours that meet your goal.

**c. Making a Table** Make a table that gives the amount of money that you will earn for each combination of hours given in part (b).

**56.** ★ **MULTIPLE CHOICE** To compete in a piano competition, you need to perform two musical pieces whose combined duration is no greater than 15 minutes. Which inequality describes the possible durations $x$ and $y$ (in minutes) of the pieces?

Ⓐ $x + y < 15$ Ⓑ $x + y \leq 15$ Ⓒ $x + y > 15$ Ⓓ $x + y \geq 15$

**57.** **MULTI-STEP PROBLEM** You are making muffins and loaves of bread for a bake sale. You need $\frac{1}{6}$ batch of batter per muffin and $\frac{1}{2}$ batch of batter per loaf of bread. You have enough ingredients to make up to 12 batches of batter.

**a.** Write and graph an inequality that describes the possible combinations of muffins $m$ and loaves $\ell$ of bread that you can make.

**b.** You make 4 loaves of bread. What are the possible numbers of muffins that you can make?

**58.** **NUTRITION** A nutritionist recommends that the fat calories $y$ consumed per day should be at most 30% of the total calories $x$ consumed per day.

**a.** Write and graph an inequality that relates the number of fat calories consumed to the total calories consumed.

**b.** Use the nutrition labels below. You normally consume 2000 calories per day. So far today you have eaten 6 crackers and 1 container of yogurt. What are the possible additional fat calories that you can consume today?



**59.** ★ **SHORT RESPONSE** You need to bring a duffel and a bedroll for a trip in the mountains. The sum of the weight $x$ (in pounds) of the duffel and the weight $y$ (in pounds) of the bedroll cannot exceed 30 pounds.

**a. Graph and Apply** Write and graph a linear inequality that describes the possible weights of the duffel and bedroll. Then give three possible combinations of weights of the duffel and bedroll.

**b. Interpret** Are (0, 30) and (30, 0) solutions of the inequality in part (a)? Do these ordered pairs make sense for this situation? *Explain.*

**60.** ★ **EXTENDED RESPONSE** A financial advisor suggests that if a person is an aggressive investor, the percent $y$ of money that the person invests in stocks should be greater than the difference of 110 and the person's age $x$.

**a. Graph** Write and graph a linear inequality that relates the percent of money invested in stocks to an aggressive investor's age.

**b. Calculate** If an aggressive investor is 30 years old, what are the possible percents that the investor can invest in stocks? *Explain* your answer.

**c. Justify** Are there any ages for which none of the solutions of the inequality makes sense for this situation? *Justify* your answer.

**61.** **CHALLENGE** The formula $m = dV$ gives the mass $m$ of an object in terms of the object's density $d$ and its volume $V$. Water has a density of 1 gram per cubic centimeter. An object immersed in water will sink if its density is greater than the density of water. An object will float in water if its density is less than the density of water.

**a.** For an object that sinks, write and graph an inequality that relates its mass (in grams) to its volume (in cubic centimeters). For an object that floats, write and graph an inequality that relates its mass (in grams) to its volume (in cubic centimeters).

**b.** A cylindrical can has a radius of 5 centimeters, a height of 10 centimeters, and a mass of 2119.5 grams. Will the can sink or float in water? *Explain* your answer.

## WISCONSIN MIXED REVIEW

**TEST PRACTICE** at classzone.com

**62.** $\triangle FGH$ has vertices $F(-2, 3)$, $G(-1, 2)$, and $H(-2, -1.5)$. $\triangle FGH$ is dilated by a scale factor of 2 and has the origin as the center of dilation. What are the coordinates of $H'$?

Ⓐ $(-4, -3)$ Ⓑ $(-4, 6)$ Ⓒ $(-1.5, -2)$ Ⓓ $(0, 0.5)$

**63.** What is the $y$-intercept of the function $f(x) = 4(x - 3)$?

Ⓐ $-12$ Ⓑ $-3$ Ⓒ $3$ Ⓓ $4$

## QUIZ for Lessons 6.5–6.7

**Solve the equation.** *(p. 390)*

**1.** $|x| = 5$ **2.** $|c - 8| = 24$ **3.** $-2|r - 5| = -6$

**Solve the inequality. Graph your solution.** *(p. 398)*

**4.** $|y| > 4$ **5.** $|2t - 5| < 3$ **6.** $4|3s + 7| - 5 \geq 7$

**Graph the inequality.** *(p. 405)*

**7.** $x + y \geq 3$ **8.** $\frac{5}{7}x < 10$ **9.** $2y - x \leq 8$



**EXTRA PRACTICE** for Lesson 6.7, p. 943 **ONLINE QUIZ** at classzone.com

## Lessons 6.5–6.7

1. **FOOD PREPARATION** You and your friends have picked 360 apples at an orchard and plan to use them to create apple pies and applesauce. You use 7 apples to make an apple pie and 5 apples to fill a jar of applesauce. Which inequality can you use to find the possible numbers $p$ of apple pies and jars $s$ of applesauce that you and your friends can make?

   (A) $5p + 7s < 360$

   (B) $7p - 5s \leq 360$

   (C) $7p + 5s \leq 360$

   (D) $360 - 7p \leq 5s$

2. **ELECTION POLL** A poll taken before an election predicts that candidate A will receive 47 percent of the vote with an absolute deviation of at most 4 percent. Which of the following equations can you use to find the minimum and maximum percent of the vote that candidate A is predicted to receive in the election?

   (A) $|47 - 4| = x$

   (B) $|47 - x| = 4$

   (C) $|x - 4| = 47$

   (D) $|47 + x| = 4$

3. **JOB TRAINING** You are scooping ice cream as part of your training to work at an ice cream shop. The weight of a scoop should be 4 ounces with an absolute deviation of at most 0.5 ounce. Your first 10 scoops have the following weights (in ounces): 3.8, 4.2, 3.9, 4.5, 3.7, 4.6, 4.1, 3.3, 4.3, and 4.2. What percent of your scoops meet the weight requirement?

   (A) 60%

   (B) 70%

   (C) 80%

   (D) 90%

4. **INVESTING** An investor purchases shares of a stock for \$30 each and will sell them if the absolute deviation of the selling price from the purchase price is at least \$15. Which inequality can you use to find the possible prices $y$ (in dollars) at which the shares will be sold?

   (A) $|y - 15| \geq 30$

   (B) $|y - 30| \geq 15$

   (C) $|y - 30| \leq 15$

   (D) $|y - 15| \leq 30$

5. **PRESENTATION** You will be making a presentation for your history class. Your teacher gives you a time limit of 15 minutes with an absolute deviation of at most 1.5 minutes. What is the minimum duration (in minutes) for your presentation?

   (A) 13.5 minutes

   (B) 15 minutes

   (C) 16.5 minutes

   (D) 22.5 minutes

6. **CONSTRUCTED RESPONSE** A tour operator recommends that a river rafter wear a protective suit under the temperature conditions described below.

| Air Temperature + Water Temperature $< 120°F$ |
|---|
| Protective suit *recommended.* |

| Air Temperature + Water Temperature $< 100°F$ |
|---|
| Protective suit *required.* |

   Write and graph an inequality that describes the possible air temperatures and water temperatures for which a protective suit is recommended.

   If the water temperature is 40°F, for which air temperatures is a protective suit recommended?

   How would you change the graph in order to describe the situations in which a protective suit is required? *Explain* your answer.